



## Nonlinear Analysis With Simple Examples

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## Outline of Presentation

- Why Nonlinear Analysis
- OpenSees Analysis Options in More Depth
- Nonlinear Beam-Column Modeling & Examples

## Why Nonlinear Analysis

- **Geometric Nonlinearities** - occur in model when applied load causes large displacement and/or rotation, large strain, or a combo of both
- **Material nonlinearities** - nonlinearities occur when material stress-strain relationship depends on load history (plasticity problems), load duration (creep problems), temperature (thermoplasticity), or combo of all.
- **Contact nonlinearities** - occur when structure boundary conditions change because of applied load.

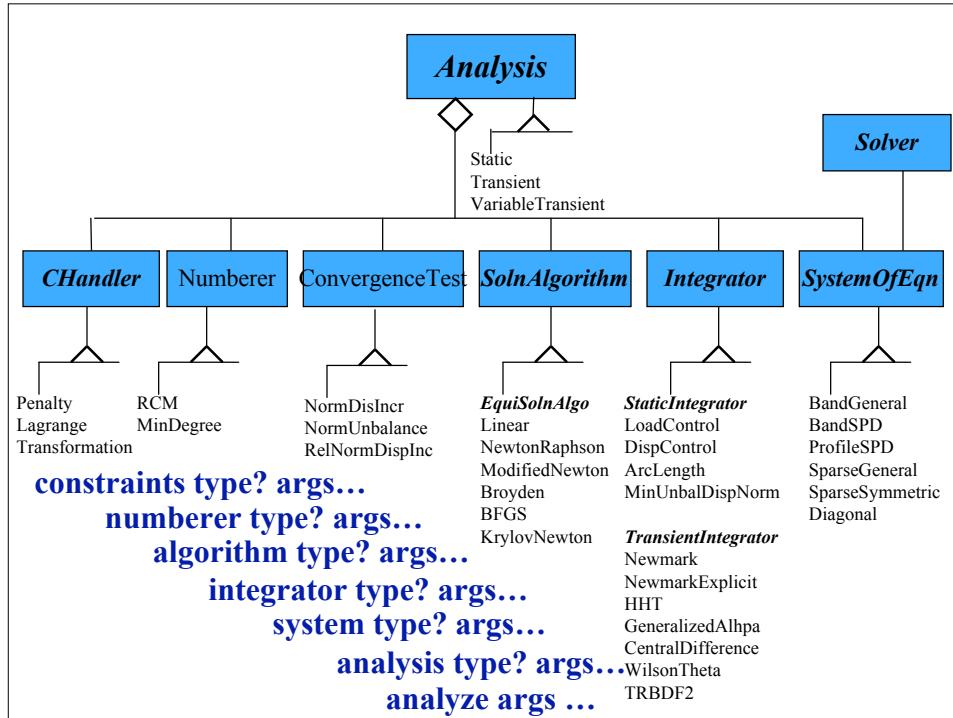
## Nonlinear Analysis is Harder

- It requires **much** more thought when setting up the model
- It requires more thought when setting up the analysis
- It takes more computational time.
- It does not always converge.
- It does not always converge to the correct solution.

**BUT Most Problems Require  
Nonlinear Analysis**

Remember that nonlinear  
analysis does not always  
converge

## CHECK YOUR MODEL



## test command:

- to specify when convergence has been achieved

all look at system: **KU = R**

- Norm Unbalance

$\sqrt{R^T R} < tol$       *test NormUnbalance tol? numIter? <flag?>*

- Norm Displacement Increment

$\sqrt{U^T U} < tol$       *test NormDispIncr tol? numIter? <flag?>*

- Norm Energy Increment

$\frac{1}{2} (U^T R) < tol$       *test NormEnergyIncr tol? numIter? <flag?>*

- Relative Tests

*test RelativeNormUnbalance tol? numIter? <flag?>*

*test RelativeNormDispIncr tol? numIter? <flag?>*

*test RelativeNormEnergyIncr tol? numIter? <flag?>*

## numberer command:

- to specify how the degrees of freedom are numbered

- Plain Numberer

nodes are assigned dof arbitrarily

*numberer Plain*

- RCM Numberer

nodes are assigned dof using the  
Reverse Cuthill-McKee algorithm

*numberer RCM*

- AMD Numberer

nodes are assigned dof using the  
Approx. MinDegree algorithm

*numberer AMD*

- numbering has an impact on performance of banded and profile solvers.  
The sparse solvers all use their own optimal numbering schemes.

## integrator command:

- determines the predictive step for time  $t+\delta t$
- specifies the tangent matrix and residual vector at any iteration
- determines the corrective step based on  $\Delta U$

- Transient Integrators for Use in Transient Analysis

Nonlinear equation of the form:

$$\mathbf{R}(\mathbf{U}, \dot{\mathbf{U}}, \ddot{\mathbf{U}}) = \mathbf{P}(t) - \mathbf{F}_l(\ddot{\mathbf{U}}) - \mathbf{F}_r(\mathbf{U}, \dot{\mathbf{U}})$$

- CentralDifference

*integrator CentralDifference*

- Newmark Method

*integrator Newmark  $\gamma$   $\beta$*

- Hilbert-Hughes-Taylor Method (alpha between 0.5 and 1.0)

*integrator HHT  $\alpha$  < $\gamma$   $\beta$ >*

- Alpha Operator Splitting Method

*integrator AlphaOS  $\alpha$*

- Static Integrators for Use in Static Analysis

Nonlinear equation of the form:

$$\mathbf{R}(\mathbf{U}, \lambda) = \lambda \mathbf{P}^* - \mathbf{F}\mathbf{R}(\mathbf{U})$$

- Load Control

$$\lambda_n = \lambda_{n-1} + \Delta\lambda \quad \text{integrator LoadControl } \Delta\lambda$$

\*does not require a reference load, i.e. loads in load patterns with Linear series and all other loads constant.

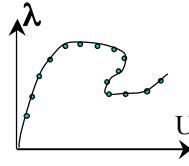
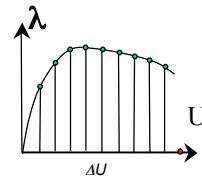
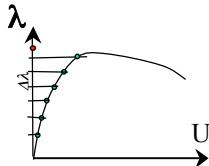
- Displacement Control

$$U_j^n = U_j^{n-1} + \Delta U_j$$

*integrator DisplacementControl node dof  $\Delta U$*

- Arc Length

$$\Delta U_n \wedge \Delta U_n + \dot{\alpha} \Delta \lambda_n = \Delta s^2 \quad \text{integrator ArcLength } \alpha \Delta s$$



## algorithm command:

- to specify the steps taken to solve the nonlinear equation

- Linear Algorithm

*algorithm Linear*

```
theIntegrator->formUnbalance();
theIntegrator->formTangent();
theSOE->solve()
theIntegrator->update(theSOE->getX());
```

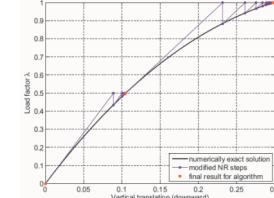
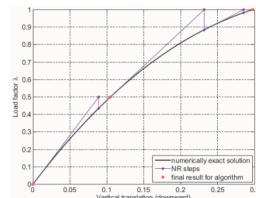
- Newton-Raphson Algorithm

*algorithm Newton*

```
theIntegrator->formUnbalance();
do {
    theIntegrator->formTangent();
    theSOE->solve()
    theIntegrator->update(theSOE->getX());
    theIntegrator->formUnbalance();
} while (theTest->test() == fail)
```

- Modified Newton Algorithm

*algorithm ModifiedNewton <-initial>*



- Accelerated Modified Newton Algorithm

*algorithm KrylovNewton <-initial>*

## constraints command:

- to specify how the constraints are enforced

$$\mathbf{U}_c = \mathbf{C}_{rc} \mathbf{U}_r$$

$$\mathbf{C} \mathbf{U} = \mathbf{0}$$

$$\mathbf{T} \mathbf{U}_r = [\mathbf{U}_r \ \mathbf{U}_c]^\wedge$$

$$[\mathbf{C}_r \ \mathbf{C}_c]^\wedge [\mathbf{U}_r \ \mathbf{U}_c] = \mathbf{0}$$

- Transformation Handler

$$\mathbf{K}^* \mathbf{U}_r = \mathbf{R}^* \quad \mathbf{K}^* = \mathbf{T}^\wedge \mathbf{K} \mathbf{T}$$

*constraints Transformation*

$$\mathbf{R}^* = \mathbf{T}^\wedge \mathbf{R}$$

in OpenSees currently don't allow retained node in one constraint to be a constrained node in another constraint

- Lagrange Handler

$$\begin{bmatrix} \mathbf{K} & \mathbf{C}^\wedge \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{Q} \end{bmatrix}$$

*constraints Lagrange*

- Penalty Handler

$$[\mathbf{K} + \mathbf{C}^\wedge \alpha \mathbf{C}] \mathbf{U} = [\mathbf{R} + \mathbf{C}^\wedge \alpha \mathbf{Q}] \quad \text{constraints Penalty } \alpha_{sp?} \alpha_{mp?}$$

## system command:

- to specify how matrix equation  $\mathbf{KU} = \mathbf{R}$  is stored and solved

- Profile Symmetric Positive Definite (SPD)



*system ProfileSPD*

- Banded Symmetric Positive Definite



*system BandSPD*

- Sparse Symmetric Positive Definite



*system SparseSPD*

- Banded General



*system BandGeneral*

- Sparse Symmetric

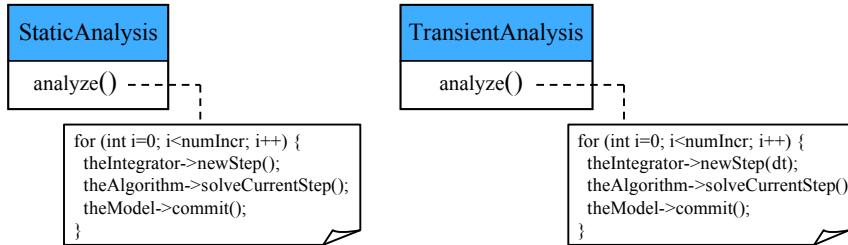


*system SparseGeneral*

*system Umfpack*

## analysis command:

- Static Analysis
- Transient Analysis
  - both incremental solution strategies



- Eigenvalue
  - general eigenvalue problem  
 $(K - \lambda M)\Phi = 0$
  - standard eigenvalue problem  
 $(K - \lambda)\Phi = 0$

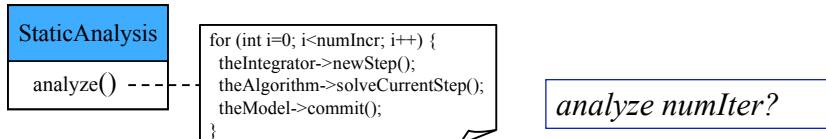
*eigen numModes? -general*

*eigen numModes? -standard*

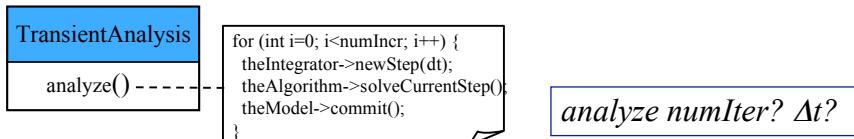
## analyze command:

- to perform the static/transient analysis

- Static Analysis



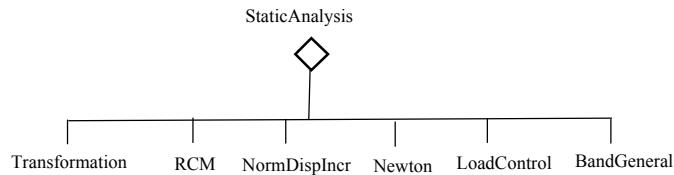
- Transient Analysis



## Example Static Analysis:

- Static Nonlinear Analysis with LoadControl

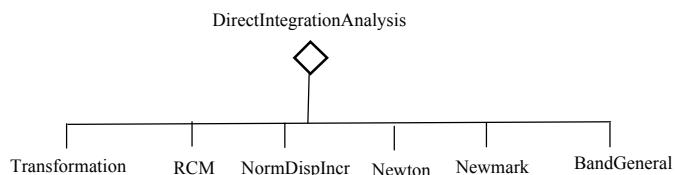
```
constraints Transformation  
numberer RCM  
system BandGeneral  
test NormDispIncr 1.0e-6 6 2  
algorithm Newton  
integrator LoadControl 0.1  
analysis Static  
analyze 10
```



## Example Dynamic Analysis:

- Transient Nonlinear Analysis with Newmark

```
constraints Transformation  
numberer RCM  
system BandGeneral  
test NormDispIncr 1.0e-6 6 2  
algorithm Newton  
integrator Newmark 0.5 0.25  
analysis Transient  
analyze 2000 0.01
```



Remember that nonlinear  
analysis does not always  
converge

## CHECK YOUR MODEL

## Commands that Return Values

- analyze command

The analyze command returns 0 if successful.  
It returns a negative number if not

```
set ok [analyze numIter <Δt>]
```

- getTime command

The getTime command returns pseudo time in Domain.

```
set currentTime [getTime]
```

- nodeDisp command

The nodeDisp command returns a nodal displacement.

```
set disp [nodeDisp node dof]
```

## Example Usage – Displacement Control

```
set maxU 15.0; set dU 0.1
constraints transformation
numberer RCM
system BandGeneral
test NormDispIncr 1.0e-6 6 2
algorithm Newton
integrator DispControl 3 1 $dU
analysis Static
set ok 0
set currentDisp 0.0
while {$ok == 0 && $currentDisp < $maxU} {
    set ok [analyze 1]
    if {$ok != 0} {
        test NormDispIncr 1.0e-6 1000 1
        algorithm ModifiedNewton -initial
        set ok [analyze 1]
        test NormDispIncr 1.0e-6 6 2
        algorithm Newton
    }
    set currentDisp [nodeDisp 3 1]
}
```

## Example Usage – Transient Analysis

```

set tFinal 15.0;
constraints Transformation
numberer RCM
system BandGeneral
test NormDispIncr 1.0e-6 6 2
algorithm Newton
integrator Newmark 0.5 0.25
analysis Transient
set ok 0
set currentTime 0.0
while {$ok == 0 && $currentTime < $tFinal} {
    set ok [analyze 1 0.01]
    if {$ok != 0} {
        test NormDispIncr 1.0e-6 1000 1
        algorithm ModifiedNewton -initial
        set ok [analyze 1 0.01]
        test NormDispIncr 1.0e-6 6 2
        algorithm Newton
    }
    set currentTime [getTime]
}

```

# Still Not Working!

1. Search the Message Board
  2. Post Problem on the Message Board

To check which scale of elcentro earthquake makes the SDF inelastic, the following file was used. By trial and error, scale 10 was found in OpenSees, which is inconsistent with the results(scale 4) from using other programs.

Is there anything wrong in this file?

```
# create ModelBuilder (with two-dimensions and 2 DOF/node)
model BasicBuilder -ndm 1 -ndf 1
```

```
# Define geometry for model  
# -----  
puts "Define geometry for mo  
set k1 2.75  
set uy 1.35
```

i suggest you check your other input files .. if you have a look at chopra's book he plots the response spectrum for this e.g. ... for a period of 0.1, D for an elastic system is with 0% damping is about .11 (fig 6.8.1 in my version) .. so you need a scale factor of about 12 [1.35/.11] to reach the ultimate.  
(note using Newmark 0.5 0.25 you get .11)

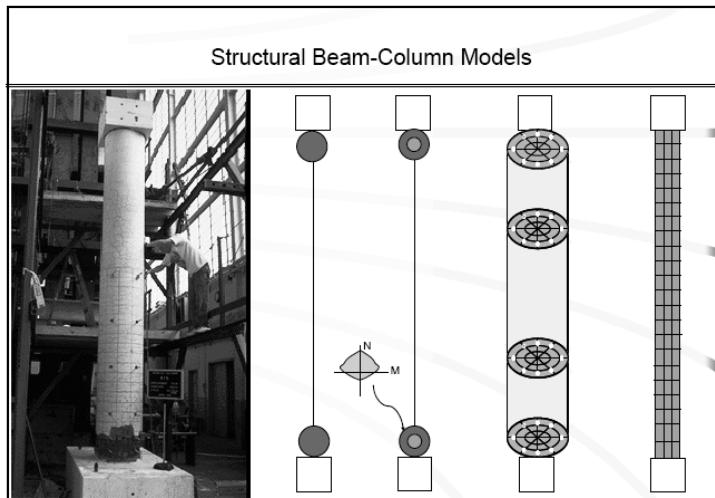
to compute the scale factor for yield i suggest you also stop playing with trying to predict the scale factor & just divide yield disp by the max response from elastic system.

## Segmentation Faults, etc:

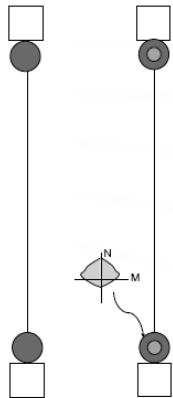
- Email: [fmckenna@ce.berkeley.edu](mailto:fmckenna@ce.berkeley.edu)

NOTE: Zip up your files in **1** directory and send them to us

## Nonlinear Beam Column Modeling



## Concentrated Plasticity Models



Advantages:

- Simple
- Good for Interface Effects (bar pullout, shear sliding)

Disadvantages:

- Properties of springs depend on geometry & Moment distribution
- Force-Displacement relationship of element needs to be related to Plastic Hinge Length & elastic element
- Properties of elastic element

```
# create modelbuilder
model BasicBuilder -ndm 2 -ndf 3;
# create 10 nodes
node 1 0.0 0.0
node 2 360.0 0.0
node 3 0.0 180.0
node 4 360.0 180.0
node 11 0.0 0.0
node 22 360.0 0.0
node 331 0.0 180.0
node 332 0.0 180.0
node 441 360.0 180.0
node 442 360.0 180.0
# Single point constraints
fix 1 1 1
fix 2 1 1
fix 11 1 0
fix 22 1 0
# Multi point constraints
equalDOF 3 331 1 2
equalDOF 3 332 1 2
equalDOF 4 441 1 2
equalDOF 4 442 1 2
# Define Materials
uniaxialMaterial Hysteretic 1 ...
uniaxialMaterial Hysteretic 2 ...
# define geometric transformation
geomTransf Linear 1
# Define Elements -----
# elastic elements:
element elasticBeamColumn 1 11 331 109.0 29000.0 10880.0 1;
element elasticBeamColumn 2 22 441 109.0 29000.0 10880.0 1;
element elasticBeamColumn 3 332 442 41.6 29000.0 14900.0 1;
# rotational hinges:
# columns
element zeroLength 11 1 11 -mat 1 -dir 6
element zeroLength 22 2 22 -mat 1 -dir 6
element zeroLength 33 3 331 -mat 1 -dir 6
element zeroLength 44 4 441 -mat 1 -dir 6
# beam
element zeroLength 55 3 332 -mat 2 -dir 6
element zeroLength 66 4 442 -mat 2 -dir 6
```

## Distributed Plasticity



- Permits the spread of plasticity along the element
  - Four or five Gauss-Lobatto points is usually sufficient
  - If no strength degradation, converges to unique solution as  $N_p$  increases
- Allows yielding to occur at any section along the element, which is important in the presence of distributed element loads
  - Girders with high gravity loads
- Integration weights from optimality constraints for the integration of high order polynomials
  - Do not reflect plastic hinge lengths from experiments or observed structural damage
  - Non-objective localized response when  $N_p$  changes (force based)
- Computation and memory overhead when yielding does not occur on the element interior
  - Typical in frame structures where columns are in double curvature (fix - beamWithHinges)

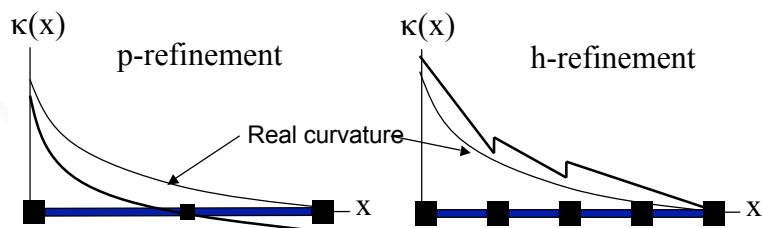
## Distributed Plasticity

### Displacement Based Formulation



element dispBeamColumn \$eleTag ...

- Constant axial deformation and linear curvature distribution enforced along the element length
  - Exact only for prismatic, linear elastic elements (Hermitian)
- Weak equilibrium leads to errors in force boundary conditions ... internal forces not in equilibrium with external reactions
- Requires  $p$ -refinement or  $h$ -refinement to represent higher order distributions of deformations



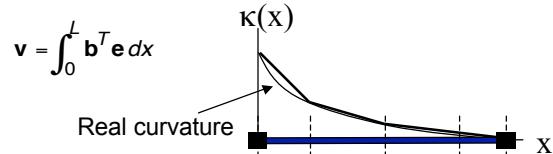
# Distributed Plasticity

## Force Based Formulation

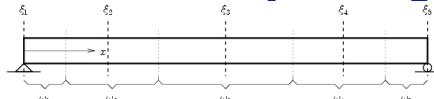
element forceBeamColumn \$eleTag ...



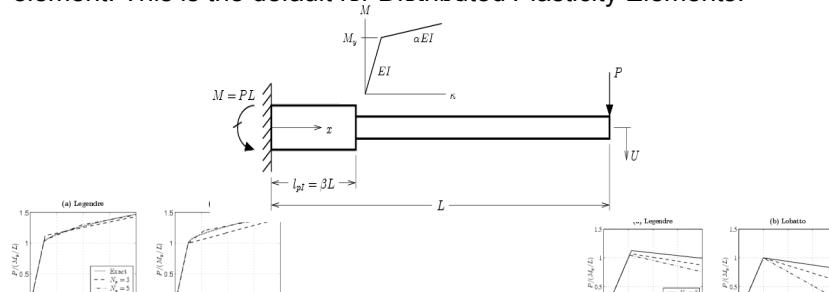
- Section forces determined from basic forces by interpolation within the basic system
  - Interpolation from static equilibrium
  - Constant axial force, linear distribution of bending moment in the absence of distributed element loads
- Equilibrium between element and section forces is exact, which holds in the range of constitutive nonlinearity
- Use Principle of Virtual Forces & Integrate section deformations along the element to get the element deformations



## Distributed Plasticity -Integration Methods



- Behaviour of element depends on choice of integration method
- Number of options: Trapezoidal, Mid-Point, and others based on gauss-quadrature (Gauss-Legendre, Gauss-Lobatto, Gauss-radau)
- Of these, only Gauss-Lobatto places integration points at ends of the element. This is the default for Distributed Plasticity Elements.



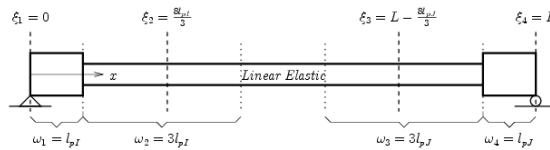
Increasing Strength

Decreasing Strength  
NON-OBJECTIVE

# BeamWithHinges Element

```
element beamWithHinges $eleTag ...
```

- The force based distributed plasticity element with inelastic sections at ends and elastic sections elsewhere, uses Gauss-Radau
- User specified hinge length.
- It is Objective even for stiffness degradation



- BUT elastic in middle

model Basic -ndm 2 -ndf 3

set ft 12.0

set in 1.0

set cm 0.3937

# Set parameters for overall model geometry

set width [expr 42.0\*\$ft]

set height [expr 36.0\*\$ft]

# Create nodes

node 1 0.0 0.0

node 2 \$width 0.0

node 3 0.0 \$height

node 4 \$width \$height

# set boundary conditions

fix 1 1 1 1

fix 2 1 1 1

# create materials for sections

uniaxialMaterial Concrete01 1 -6.0 -0.004 -5.0 -0.014; # core

uniaxialMaterial Concrete01 2 -5.0 -0.002 0.0 -0.006; # cover

uniaxialMaterial Steel01 3 60.0 30000.0 0.01

# create sections

set bWidth [expr \$width]; set bDepth [expr 5.0\*\$ft]; set cover 1.5

set As 0.60; # area of no. 7 bars

source RCsection2D.tcl

RCsection2D 1 \$bWidth \$bDepth \$cover 1 2 3 6 3 \

    \$As 10 10 2

#create geometric transformations

geomTransf PDelta 1

geomTransf Linear 2

# Create the columns usind distributed plasticity elements

set np 5

**set eleType forceBeamColumn**

**element ElecType 1 1 3 \$np 1 1**

element ElecType 2 2 4 \$np 1 1

# Create the beam element using elastic element

set d [expr 8.0\*\$ft]; # Beam Depth

set b [expr 5.0\*\$ft]; # Beam Width

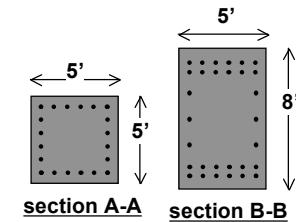
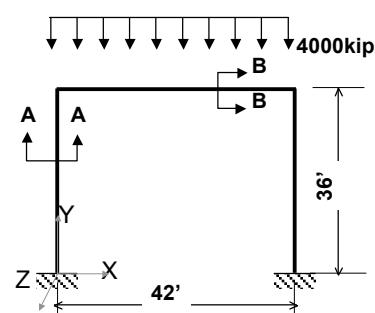
set Eb 3600.0; set Ab [expr \$b\*\$d]; set Izb [expr (\$b\*\$pow(\$d,3))/12.0];

**section Elastic 2 \$Eb \$Ab \$Izb;** # elastic beam section

#element elasticBeamColumn 3 3 4 \$Ab \$Eb \$Izb 2

element ElecType 3 3 4 \$np 2 2

## Model



RCFrameGravity.tcl

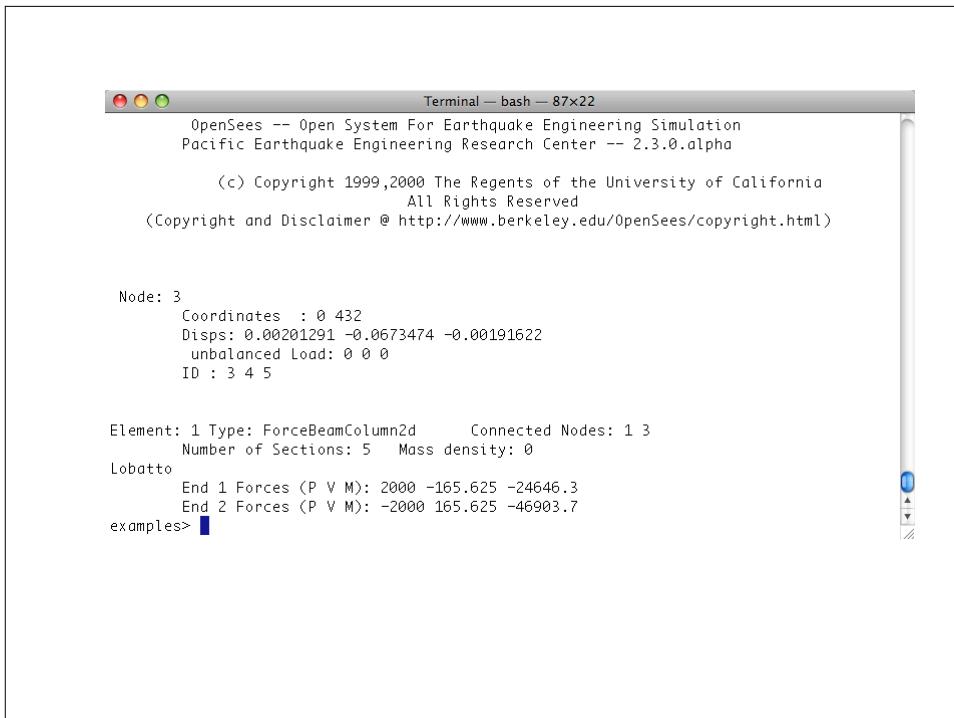
## Gravity Load Analysis

```
# first source in the model
source RCFrame.tcl

# Create the gravity loads
set W 4000.0;
timeSeries Linear 1
pattern Plain 1 1 {
    eleLoad -ele 3 -type -beamUniform [expr -$W/$width]
}

# create the analysis
system BandGeneral
constraints Transformation
numberer RCM
test NormDispIncr 1.0e-12 10 3
algorithm Newton
integrator LoadControl 0.1
analysis Static

# perform the analysis
analyze 10
```



The screenshot shows a terminal window titled "Terminal — bash — 87x22". The window displays the OpenSees software interface. At the top, it says "OpenSees -- Open System For Earthquake Engineering Simulation Pacific Earthquake Engineering Research Center -- 2.3.0.alpha". Below that, copyright information from the University of California is shown. The main area of the terminal contains the results of a command-line session. It starts with "Node: 3" followed by its coordinates and ID. Then it lists an element: "Element: 1 Type: ForceBeamColumn2d Connected Nodes: 1 3 Number of Sections: 5 Mass density: 0". The "Lobatto" section follows, showing force values at both ends. Finally, there is a prompt "examples>".

```
Terminal — bash — 87x22
OpenSees -- Open System For Earthquake Engineering Simulation
Pacific Earthquake Engineering Research Center -- 2.3.0.alpha

(c) Copyright 1999,2000 The Regents of the University of California
All Rights Reserved
(Copyright and Disclaimer @ http://www.berkeley.edu/OpenSees/copyright.html)

Node: 3
Coordinates : 0 432
Disps: 0.00201291 -0.0673474 -0.00191622
unbalanced Load: 0 0 0
ID : 3 4 5

Element: 1 Type: ForceBeamColumn2d Connected Nodes: 1 3
Number of Sections: 5 Mass density: 0
Lobatto
End 1 Forces (P V M): 2000 -165.625 -24646.3
End 2 Forces (P V M): -2000 165.625 -46903.7
examples>
```

## Pushover Analysis

```

source RCFramesGravity.tcl
puts "Gravity Analysis Completed"

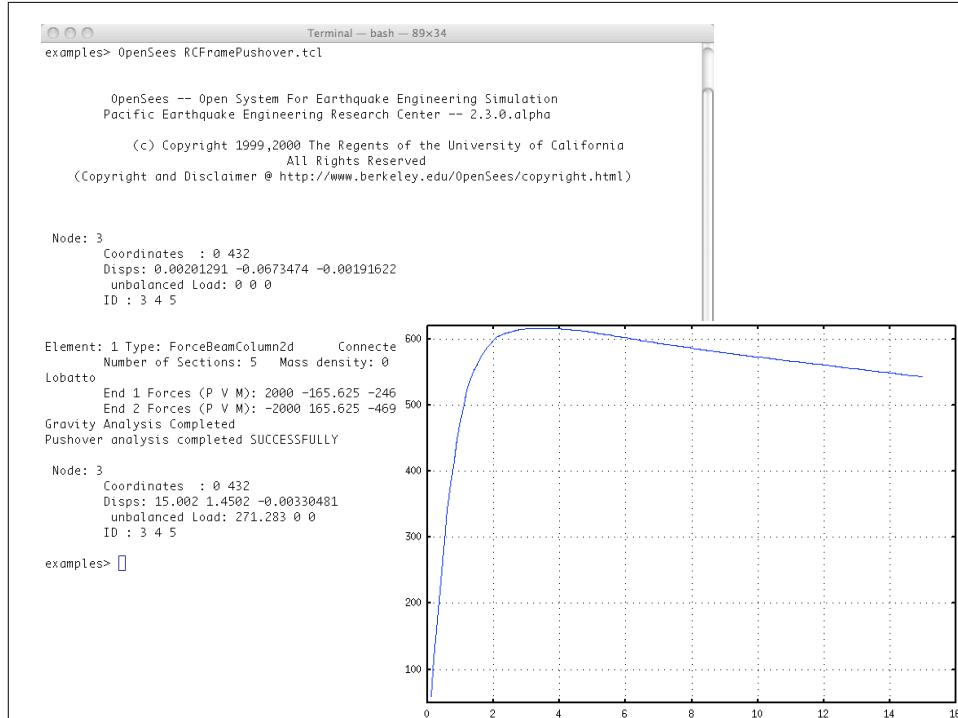
# Set the gravity loads to be constant & reset the time in the domain
loadConst -time 0.0

# Define Pattern for Lateral Reference loads
set H 10.0;
pattern Plain 2 1 {
    load 3 $H 0.0 0.0
    load 4 $H 0.0 0.0
}
set dU 0.1;
integrator DisplacementControl 3 1 $dU 1 $dU $dU

# Set some parameters
set maxU 15.0;      # Max displacement
set currentDisp 0.0;
set ok 0
while {$ok == 0 && $currentDisp < $maxU} {
    set ok [analyze 1]

    # if the analysis fails try initial tangent iteration
    if {$ok != 0} {
        puts "regular newton failed .. lets try an initail stiffness for this step"
        test NormDispIncr 1.0e-12 1000
        algorithm ModifiedNewton -initial
        set ok [analyze 1]
        test NormDispIncr 1.0e-12 10
        algorithm Newton
    }
    set currentDisp [nodeDisp 3 1]
}
if {$ok == 0} {
    puts "Pushover analysis completed SUCCESSFULLY";
}

```



## Transient Analysis - Uniform Excitation

```

source RCFraneGravity.tcl
puts "Gravity load analysis completed"
# Set the gravity loads to be constant
# & reset the time in the domain
loadConst -time 0.0

# Define nodal mass
set g 386.4
set m [expr ($W/2.0)/$g];
# tag MX MY RZ
mass 3 $m $m 1.0e-16
mass 4 $m $m 1.0e-16

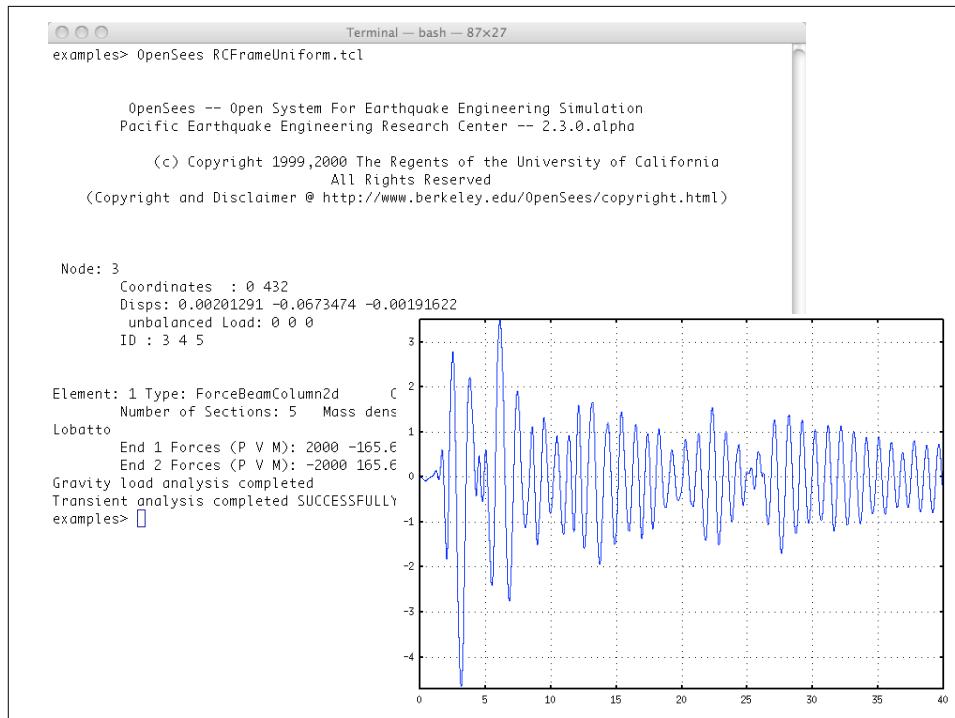
# Define dynamic loads
set record IELC180
source ReadRecord.tcl
ReadRecord $record.AT2 $record.dat dT nPts
timeSeries Path 2 -filePath $record.dat -dt $dT
pattern UniformExcitation 2 1 -accel 2
rayleigh 0.0 0.0 0.0 0.0
#create a recorder
recorder Node -time -file disp.out -node 3 4 -dof 1 2 3 disp
# remove old analysis
wipeAnalysis

#create the analysis
system BandGeneral
constraints Plain
test NormDispIncr 1.0e-8 10
algorithm Newton
numberer RCM
integrator Newmark 0.5 0.25
analysis Transient

set tFinal [expr $nPts * $dT]
set tCurrent [getTime]
set ok 0
# perform the analysis
while {$ok == 0 && $tCurrent < $tFinal} {
    set ok [analyze 1 $dT]
    # if the analysis fails try initial tangent iteration
    if {$ok != 0} {
        puts "regular newton failed .. lets try another"
        test NormDispIncr 1.0e-8 1000 1
        algorithm ModifiedNewton -initial
        set ok [analyze 1 $dT]
        test NormDispIncr 1.0e-12 10
        algorithm Newton
    }
    set tCurrent [getTime]
}

# Print a message to indicate if analysis successful
if {$ok == 0} {
    puts "Transient analysis completed FAILED";
} else {
    puts "Transient analysis completed SUCCESS";
}

```



## Transient Analysis - MultiSupport Excitation

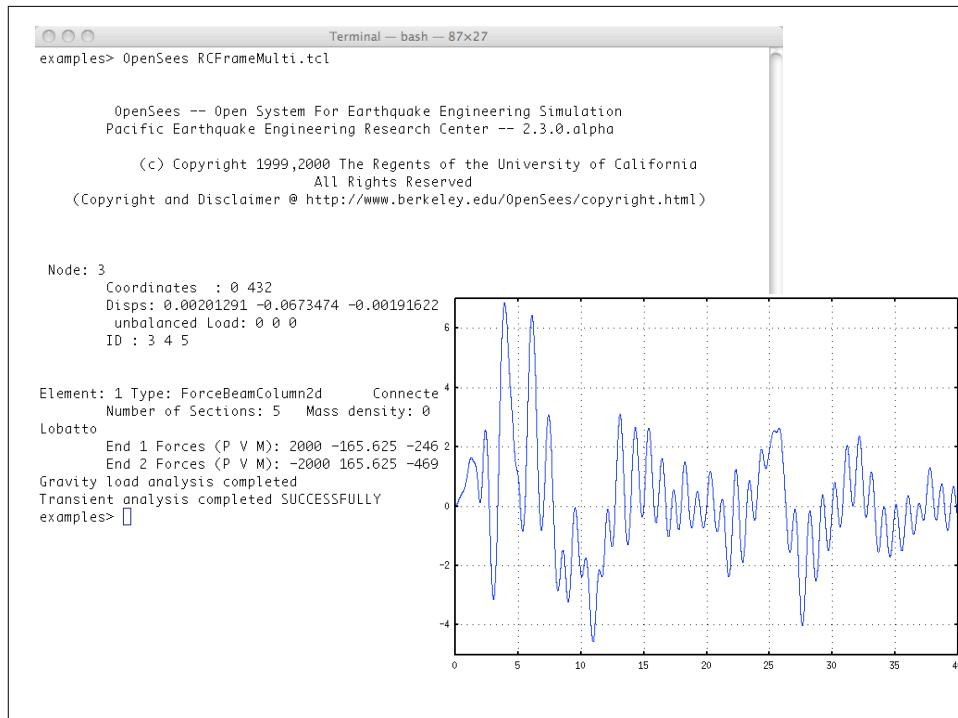
```

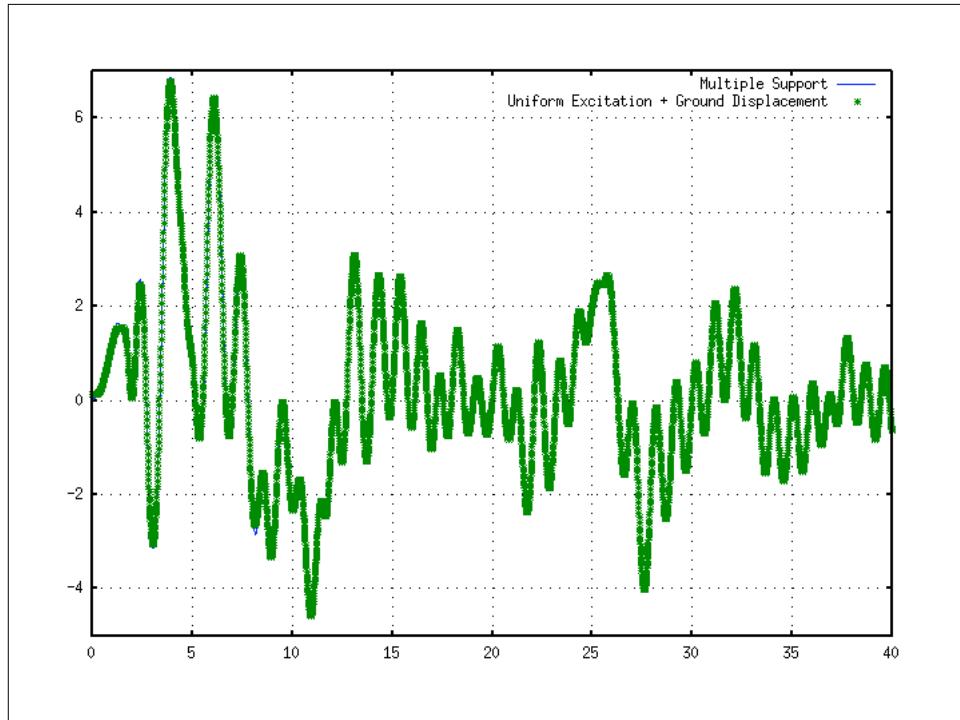
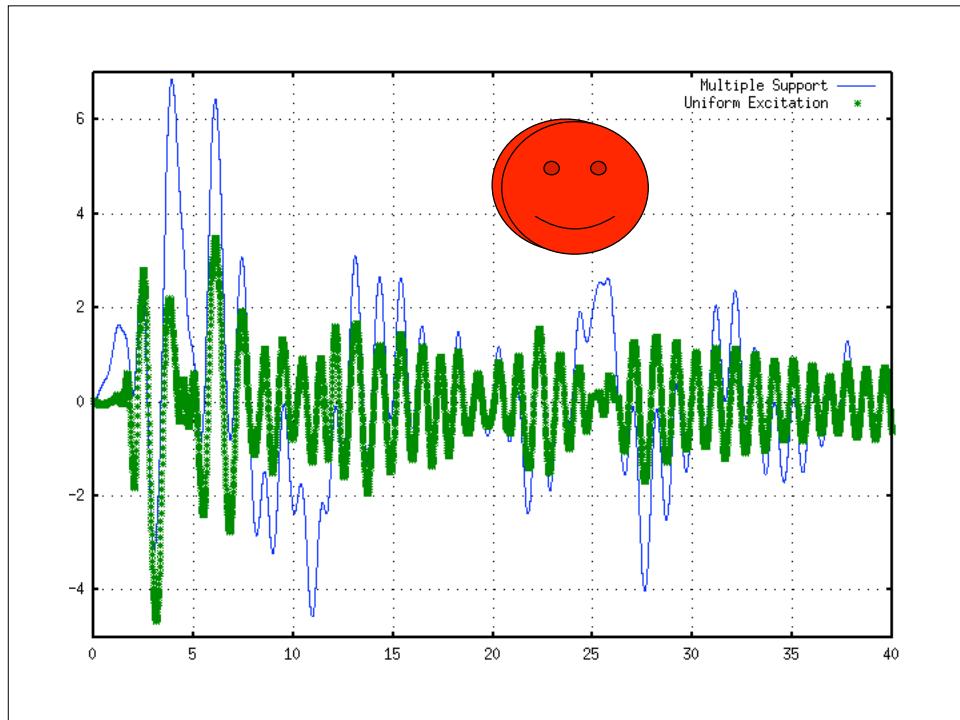
source RCFrameGravity.tcl
# Set the gravity loads to be constant
# & reset the time in the domain
loadConst -time 0.0

# Define nodal mass
set m [expr ($W/2.0)/$g];
# tag MX MY RZ
mass 3 $m $m 1.0e-16
mass 4 $m $m 1.0e-16

# Define dynamic loads
# Set some parameters
set record IELC180
# Source in TCL proc to read PEER SMD record
source ReadRecord.tcl
ReadRecord $record.DT2 $record.dat dT nPts
timeSeries Path 2 -filePath $record.dat -dt $dT -factor $cm
pattern MultiSupport 2 {
    groundMotion 5 Plain -disp 2
    imposedMotion 1 1 5
    imposedMotion 2 1 5
}
recorder Node -time -file multi.out -node 1 3 -dof 1 disp
rayleigh 0.0 0.0 0.0 0.0 0.0
remove sp 1 1
remove sp 2 1
wipeAnalysis
#create the analysis
constraints Plain
test NormDispIncr 1.0e-8 10
algorithm Newton
numberer RCM
integrator Newmark 0.5 0.25
analysis Transient
set tFinal [expr $nPts * $dT]
set tCurrent [getTime]
set ok 0
# perform the analysis
while {$ok == 0 && $tCurrent < $tFinal} {
    set ok [analyze 1 $dT]
    # if the analysis fails try initial tangent iteration
    if {$ok != 0} {
        puts "regular newton failed .. lets try another"
        test NormDispIncr 1.0e-8 1000 1
        algorithm ModifiedNewton -initial
        set ok [analyze 1 $dT]
        test NormDispIncr 1.0e-12 10
        algorithm Newton
    }
    set tCurrent [getTime]
}
# Print a message to indicate if analysis successful
if {$ok == 0} {
    puts "Transient analysis completed SUCCESS"
} else {
    puts "Transient analysis completed FAILED"
}
}

```





## Parameter Study - Response Spectra

```

source READSMDFile.tcl
modelBuilder BasicBuilder -ndm 1 -ndf 1

# set a bunch of parameters
set PI 3.14159265
set g 386.4
set TnMin 0.1; #min period
set TnMax 2.0; #max period
set TnIncr 0.1; #period incr
set M 1.0;      #mass
set A 1.0;      #area
set L 1.0;      #length
set motion ELCENTRO
set outFilename spectrum.dat

# open output file
Set outFileID [open $outFilename w]

#create accel series
ReadSMDFILE Smotion.AT2 Smotion.acc dt
Set accelSeries "Path -filePath $motion.acc \
-dt $dt -factor $g"

# loop over period range
Set Tn $TnMin
while {$Tn <= $TnMax} {
    wipe
    set w [expr 2.0 * SPI / $Tn]
    set K [expr $w * $w * $M]
    set E [expr $K * $L / $A]

    node 1 0.0
    node 2 $L -mass $M
    fix 1 1
    uniaxialMaterial Elastic 1 $E
    element truss 1 1 2 $A 1
    pattern UniformExcitation 2 1 -accel $accelSeries
    rayleigh 0.0 0.0 0.0 0.0 0.0

    recorder EnvelopeNode -file envelope.out -node 2 -dof 1 disp
    system ProfileSPD
    test NormDispIncr 1.0e-16 10
    algorithm Newton
    integrator Newmark 0.5 0.25
    analysis Transient
    analyze 2000 $dt

    if [catch {open envelope.out r} inFileID]
        puts puts "ERROR - could not open file"

    set min [gets $inFileID]
    set max [gets $inFileID]
    set absMax [gets $inFileID]
    close $inFileID
    puts $outFileID "$Tn $absMax"
    set Tn [expr $Tn + $TnIncr]
}
close $outFileID

```

```

Terminal — OpenSees — 82x38
cee-84-111:~/OpenSees/EXAMPLES/ExampleScripts/ExampleScripts fmk$ ~/bin/OpenSees

OpenSees -- Open System For Earthquake Engineering Simulation
Pacific Earthquake Engineering Research Center -- Version 1.6.0

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OpenSees > source ResponseSpectra.tcl
OpenSees > cat spectrum.dat
0.1 0.0786084
0.2 0.419001
0.3 0.753439
0.4 1.47281
0.5 2.66804
0.6 3.0994
0.7 3.37357
0.8 3.70962
0.9 6.24449
1.0 5.9645
1.1 4.9327
1.2 4.75759
1.3 3.94977
1.4 4.41569
1.5 4.72872
1.6 5.93379
1.7 6.3168
1.8 6.72183
1.9 7.40134
2.0 7.47503

```